Before and After Target Date Investing: The General Equilibrium Implications of Retirement Saving Dynamics

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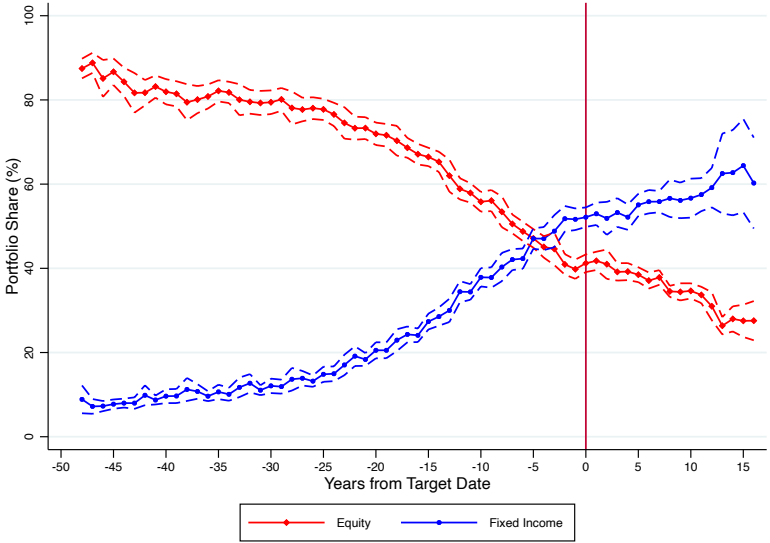
Motivation

- Household portfolios
  - limited stock market participation, especially by young and low wealth workers
  - inertia: rarely rebalance between stocks and bonds
  - stable & concentrated market shares: top 10% account for 80% of holdings 1989-2019
- Recent innovation: Target date funds
  - portfolio share on stocks is initially high, declines with age

Target Date Funds - Glide Path



Motivation

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  - limited stock market participation, especially by young and low wealth workers
  - inertia: rarely rebalance between stocks and bonds
  - stable & concentrated market shares: top 10% account for 80% of holdings 1989-2019
- Recent innovation: Target date funds
  - portfolio share on stocks is initially high, declines with age
  - commonly used as the default investment (76% all plans, PSCA 2021)
  - assets under management: 8 billion (2000) to 3.3 trillion (2021)
- Effects of limited access to equity markets for asset prices, inequality, and welfare?

This paper

- Overlapping generations model with idiosyncratic and aggregate risk
  - solve using machine learning to overcome the curse of dimensionality
- Benchmark economy: frictions in stock market participation & rebalancing
  - quantified using portfolio data 1995-2001 (before target date funds)
  - matches macro aggregates, equity returns and riskfree rate, portfolio distribution
  - generate inelastic stock demand, concentrate equity holdings
- Target date economy: close to a world without the frictions
  - asset prices: equity premium 6% to 2%, equity return volatility 22% to 14%
  - inequality: top 10% stock market share drops from 77% to 53%
  - welfare through wealth redistribution: 20-30% lifetime consumption equivalents for bottom 90% of wealth distribution at the expense of top 10%
- Free access economy: remove both frictions, similar outcomes to target date economy

Literature

- Household portfolio
  - empirical evidence for non-participation and inertia:
    Mankiw-Zeldes 91, Poterba-Samwick 95, Vissing-Jorgensen 98, Madrian-Shea 01,
    Choi-Laibson-Madrian-Metrick 02, Agnew-Balduzzi-Sundén 03, Ameriks-Zeldes 04,
    Beshears-Choi-Laibson-Madrian 09, Brunnermeier-Nagel 08, Calvet-Campbell-Sodini 09, Mitchell-Utkus 21,
    Parker-Schoar-Cole-Simester 22, Balloch-Richers 2023, Gabaix-Koijen-Mainardi-Oh-Yogo 2023
  - portfolio choice with target date funds:
    Gomes-Kotlikoff-Viceira 08, An-Sachdeva 21, Duarte-Fonseca-Goodman-Parker 21,
    Gomes-Michaelides-Zhang 22
- Equilibrium models with limited access to equity markets

  Allen-Gale 94, Heaton-Lucas 96, Gabaix-Laibson 02, Guvenen 09, Chien-Cole-Lustig 12
- Stock prices in OLG economies

  Abel 03, Geanakoplos-Magill-Quinzii 04, Storesletten-Telmer-Yaron 07, Favilukis 13, Garleanu-Panageas 15,
  Leombroni-Piazzesi-Roger-Schneider 20
- Machine learning tools for computing models with aggregate risk

  Kahou-Fernandez-Villaverde-Perla-Sood 21, Maliar-Maliar-Winant 21, Azinovic-Gaegauf-Scheidegger 22,
  Han-Yang-E 22, Payne-Rebei-Yang 2024

Outline

- OLG model with idiosyncratic and aggregate risk
- Machine learning solution
- Quantification of the model
- Benchmark results
- Counterfactuals: target date and free access economies

OLG model

- Continuum of households
  - finite horizon with stochastic death arrival
  - discount future at rate ρ, CRRA utility, utility from bequest
- Aggregate state
  - persistent Markov chain $Z_t \in \{0, 1\}$
- Income dynamics
  - deterministic age profile
  - idiosyncratic risk correlated with aggregate state
  - constant social security payment for retirees

Frictions in participation and portfolio rebalancing BM AT

- Stock market participation shocks
  - arrival rate correlated with income
  - some agents receive bequests and participation shocks at the beginning of life cycle
  - choose contribution allocation rule subject to short selling constraints
  - withdraw in proportion to portfolio shares
- Setup captures three features of household portfolios
  - explicit contribution rule: majority of households participate in financial markets through retirement accounts
  - limited participation: non-participants have a contribution rule that is 0% in stocks
  - wealthy households: flows insensitive to stock returns (Gabaix et al. 2023)

Frictions in participation and portfolio rebalancing BM AT

- Stock market participation shocks
  - Poisson arrival rate: 0 for low, ∞ for star, $\lambda(age)$ for high
  - some agents receive bequests and participation shocks at the beginning of life cycle
  - choose contribution allocation rule subject to short selling constraints
  - withdraw in proportion to portfolio shares
- Deviation from standard consumption based asset pricing models
  - consume and save freely, taking portfolio weights as given
  - Euler conditions do not hold for individual assets
  - instead, hold for returns on portfolios
- Alternative asset market arrangements
  - target date economy: portfolio weights are on the glide path
  - free access economy: freely choose portfolio allocation

Rest of the economy

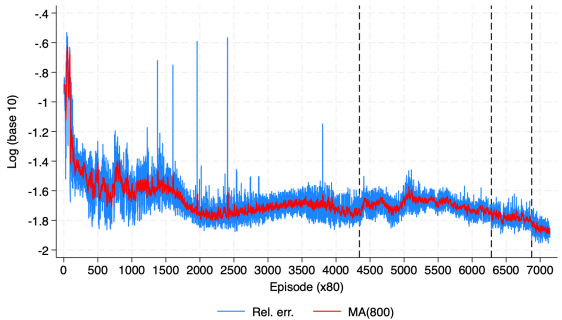
- A continuum of firms
  - Cobb-Douglas production technology
  - issue riskfree debt to finance investment in risky capital
  - investment adjustment cost
  - implement payout and capital structure rules
    - micro-founded with maximizing discounted log payouts
- Government
  - collects income taxes, pays for social security, issues government bonds
  - adjusts discretionary spending to balance its budget

Equilibrium GD

- Recursive competitive equilibrium
- Challenge
  - distribution of individual state variables is a state variable!
  - age, income state, equity, bond holdings, contribution rule
    $\Longrightarrow$ need a feasible and sensible representation
  - 5 individual state variables, 2 other aggregate state variables

Machine learning solution

- DeepHAM: AI as an agent (Han-Yang-E 2022)
  - fictitious play between model environment and agent being trained
  - neural nets for policies and generalized moments
  - automated Krusell-Smith



Relative consumption error: 1.3%

Machine learning solution

Lessons and tricks so far:

- big nets prone to NaN traps, especially before nets are well trained
- medium-sized nets for policy functions and generalized moments
- different learning rates for policy and GMs
- after convergence, check and use copies of neural nets in large error regions
- large cross sections help improve accuracy

Both strenuous on GPU memory, possible solution:

- unified memory from multiple GPUs
- (?) multi-GPU training with Julia Flux

Quantification CT

- Pre-set parameters
- Targeted aggregate wealth moments
  - household discount rate: wealth-to-income ratio
  - bequest intensity: retiree wealth share
  - bequest intercept: top 10% wealth share
- Model Fit
  - macroeconomic aggregates and asset prices
    - growth rate volatility of output, consumption, investment, labor supply
    - equity premium, equity return volatility, Sharpe ratio, leverage
  - untargeted distribution: wealth age profile
    - lifecycle saving incentive
  - untargeted distribution: equity market share by age
    - intensive margin governed by inertia
  - untargeted distribution: equity market share by wealth
    - participation and rebalancing frictions concentrate equity holdings

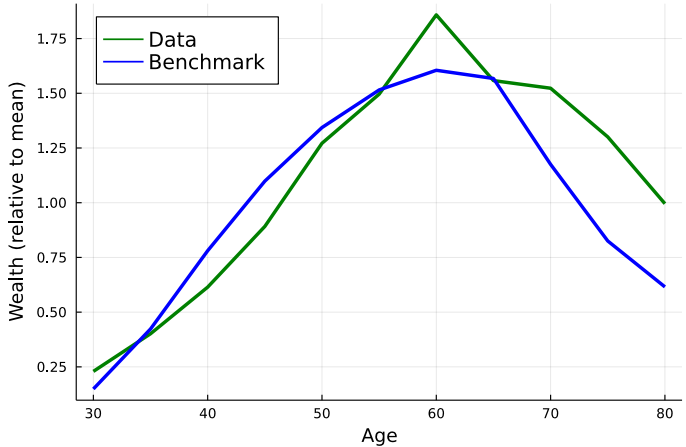
Aggregate Moments

	Quarterly SD (Growth Rate)			
	Y	C	I	L
Benchmark	0.017	0.018	0.034	0.010
Data	0.012	0.012	0.041	0.014

	Annualized Asset Returns			
	$E[r_t - r_t^f]$	$\sigma(r_t - r_t^f)$	Sharpe Ratio	leverage
Benchmark	0.063	0.247	0.254	0.572
Data	0.066	0.178	0.371	0.560

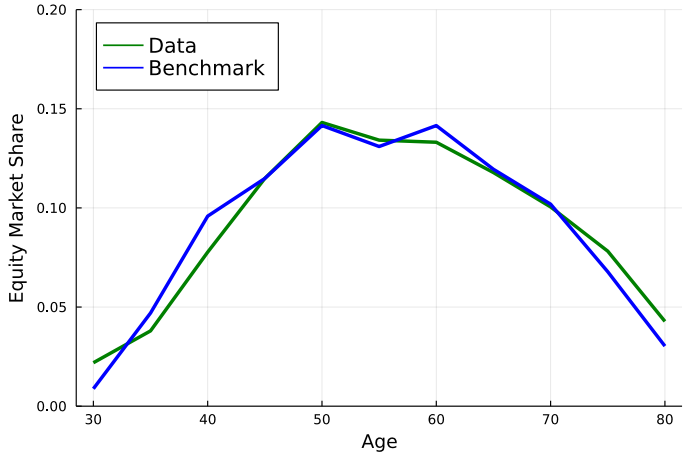
- 1970Q1-2022Q2

Untargeted Distribution: Wealth Age Profile



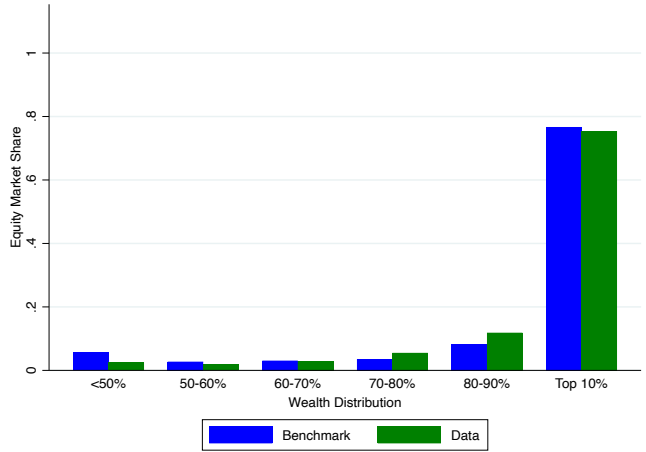
- Life-cycle saving incentives line up with data
- Hump-shape: save while working, dissave in retirement

Untargeted Distribution: Equity Market Shares by Age



- Targeted extensive margin (participation)
- Intensive margin governed by inertia

Untargeted Distribution: Equity Market Shares by Wealth



- Participation friction excludes a fraction of households from holding stocks
- Rebalancing friction concentrates even more: stock returns higher than bonds

Counterfactuals

- Two alternative asset market arrangements
  - target date economy - everyone on the glide path
  - free access economy - freely access asset markets and choose portfolio weights
- Portfolio age profile: more participation in equity market
- Asset prices: equity return is lower, less volatile
- Equity market share age <55: 53% (benchmark), 70% (target date), 60% (free access)
- Top 10% equity market share: 77% (benchmark), 53% (target date), 46% (free access)
- Sharpe ratio
  - equity shares redistribution across age cohorts: individual effect, human capital
  - equity shares redistribution across wealth distribution: compositional effect
- Welfare
  - bottom 90% better off: easier access to equity
  - top 10% worse off: reduction in equity premium

Counterfactual: Target Date Economy - Asset Prices cd

Annualized Asset Returns

	$E[r_t]$	$\sigma(r_t)$	$E[r_t^f]$	$\sigma(r_t^f)$	Sharpe Ratio	$E[r_t - r_t^f]$
Benchmark	0.064	0.219	0.000	0.007	0.292	0.064
Target Date	0.017	0.146	-0.001	0.009	0.116	0.017

- Equity premium smaller
  - product of stock return volatility and Sharpe ratio

Counterfactual: Target Date Economy - Asset Prices cd

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- Equity premium smaller
  - product of stock return volatility and Sharpe ratio
- Equity returns are less volatile
  - more elastic stock demand: trade against market outcomes

Counterfactual: Target Date Economy - Asset Prices cd

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	$E[r_t]$	$\sigma(r_t)$	$E[r_t^f]$	$\sigma(r_t^f)$	Sharpe Ratio	$E[r_t - r_t^f]$
Benchmark	0.064	0.219	0.000	0.007	0.292	0.064
Target Date	0.017	0.146	-0.001	0.009	0.116	0.017

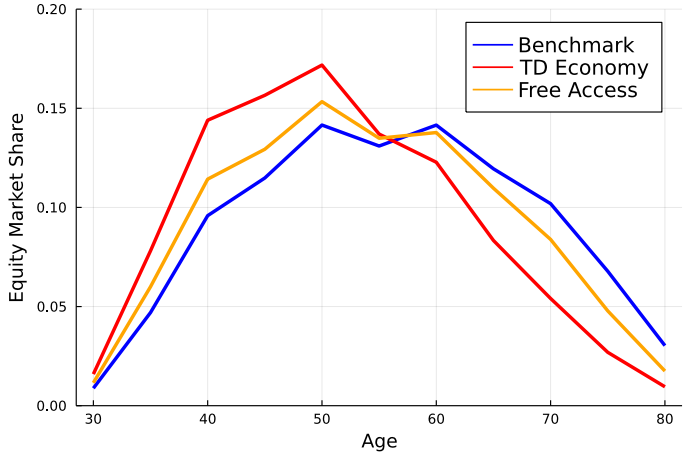
- Equity premium smaller
  - product of stock return volatility and Sharpe ratio
- Equity returns are less volatile
  - more elastic stock demand: trade against market outcomes
- Sharpe ratio down
  - redistribution of equity holdings leads to better risk sharing

Counterfactual: Free Access Economy - Asset Prices

	Annualized Asset Returns					
	$E[r_t]$	$\sigma(r_t)$	$E[r_t^f]$	$\sigma(r_t^f)$	Sharpe Ratio	$E[r_t - r_t^f]$
Benchmark	0.064	0.219	0.000	0.007	0.292	0.064
Target Date	0.017	0.146	-0.001	0.009	0.116	0.017
Free Access	0.016	0.131	0.003	0.008	0.099	0.013

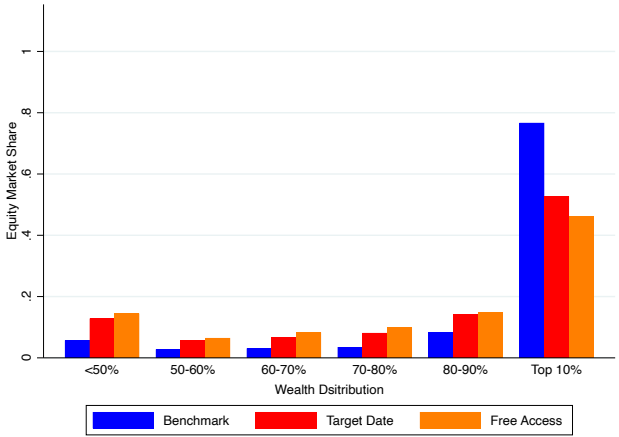
- Free access economy: same direction, further reductions
  - more elastic stock demand: trade against market outcomes
  - redistribution of equity holdings leads to better risk sharing

Counterfactual: More Equity Holdings for the Young



- Age 55 and less: 53% (benchmark), 70% (target date), 60% (free access)
- Better risk sharing: the young are rich in relatively safe human capital

Counterfactual: Reduction in the Concentration of Equity Holdings



- Top 10% equity market share: 77% (benchmark), 53% (target date), 46% (free access)
- Better risk sharing: top 10% more exposed to aggregate risk, high risk compensation

Counterfactuals

- Target date glide path: high while working low in retirement
- Equity premium is lower: 6.4% to 1.7%
  - less volatility: 22% to 14%
  - better risk sharing
- Equity market share age <55: 53% (benchmark), 70% (target date)
- Top 10% equity market share: 77% (benchmark), 53% (target date)
- Sharpe ratio
  - equity shares redistribution across age cohorts: individual effect, human capital
  - equity shares redistribution across wealth distribution: compositional effect
- Welfare
  - bottom 90% enjoy 20-30% consumption equivalents: easier access to equity
  - top 10% lose up to 30% consumption equivalents: reduction in equity premium
- Similar outcomes in free access economy

Conclusion

- Effects of limited access to equity markets for asset prices, inequality, and welfare?
- Approach
  - OLG with idiosyncratic and aggregate risk
  - generalized moments + reinforcement learning to overcome curse of dimensionality
- Findings
  - limited participation + infrequent rebalancing explain high stock volatility and premium
  - target date investing: lower risk premium and inequality, welfare gains for bottom 90%
  - target date outcomes: close to free access and full optimization
- Policy that changes retirement savings dynamics on a macro scale
  - 2006 Pension Protection Act, 2022 SECURE Act 2.0
  - large general equilibrium effects to consider

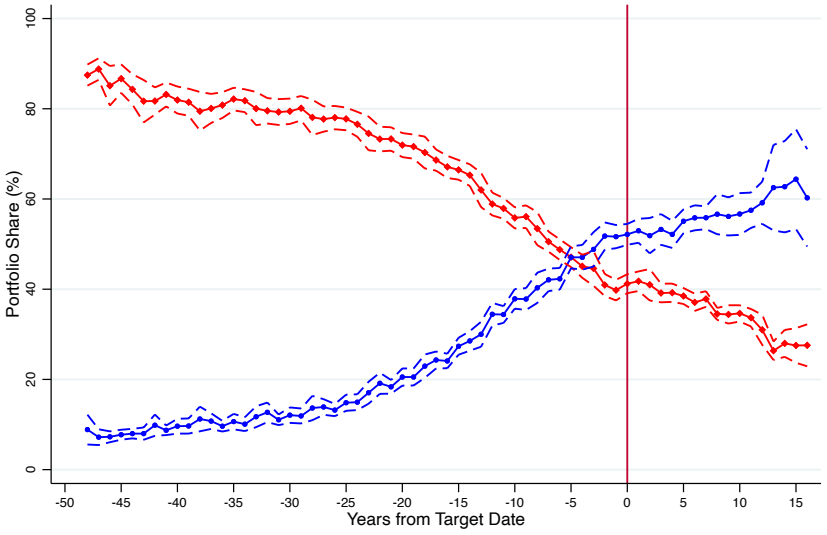
Thank You!

Appendix

Glide Path

back

- mutual funds/CITs, e.g. XXX 2060 Retirement Fund, XXX 2070 Lifecycle Fund



Household Problem back

$$V(X_t) = \sup_{c,\tilde{f}} E_t \left[\int_t^{t+a^{exit}-a_t} exp^{-\rho(u-t)-\int_t^u \eta(a_s)ds} \left(u(c_u) + \eta(a_u) u^B(q_u) \right) du \right]$$

$$c_t + s_t = \begin{cases} w_t l(a_t, y_t) & a_t \leqslant a^{retire} \\ \bar{s} & a_t > a^{retire} \end{cases}$$

$$de_t = (\mu_t^e e_t + \widetilde{f}_t s_t)dt + \sigma_t^e e_t dW_t$$

$$db_t = [r_t^f b_t + (1 - \widetilde{f}_t)s_t]dt$$

$$c_t, \tilde{f}_t, e_t, b_t \geqslant 0,$$

- $\tilde{f}_t \in [0,1]$: fraction of flow allocated to stocks

Asset Market Arrangements back

- Benchmark economy

$$\widetilde{f}_t = \begin{cases} f_t & s_t \geqslant 0 \\ \frac{e_t}{e_t + b_t} & s_t < 0 \end{cases} \qquad f_t = \begin{cases} 0 & t \leqslant T_1^f \\ F_{T_1^f} & t > T_1^f \end{cases}$$

- Target date economy: $\tilde{f}_t$ is determined so that portfolios weights are on glide path $T(a_t)$

$$\frac{e_t}{e_t + b_t} = T(a_t)$$

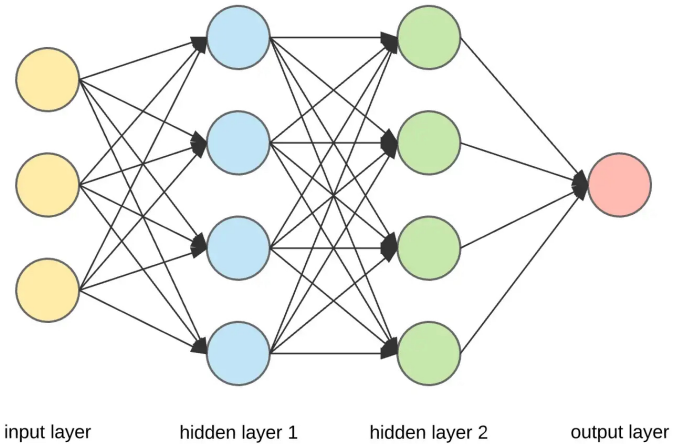
- Free access economy: freely choose $\tilde{f}_t$

Government Details SIFMA back

- Collects income taxes at a constant tax rate
- Pays for social security, interest on government bonds, and discretionary spending
- Government bond supply makes up a fixed fraction of the total bond market
- Balances budget by adjusting discretionary spending

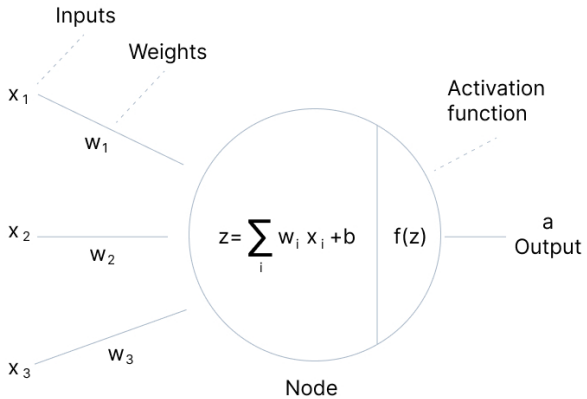
Artificial Neural Network: An Adjustable Function

neuron back



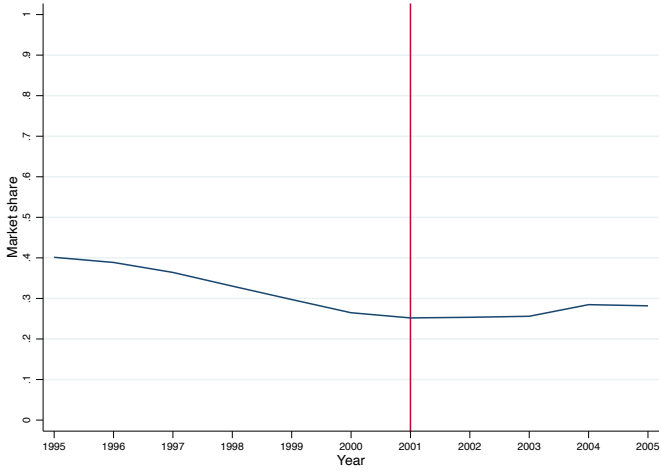
- Example: consumption function

Artificial Neuron back



Source: Baheti 2022

Government Bond Market Share back



Asset Prices

back

		Annualized Asset Return and Standard Deviation					
		Equity Return		Riskfree Rate		Equity Premium	
		$E[r_t]$	$\sigma(r_t)$	$E[r_t^f]$	$\sigma(r_t^f)$	$E[r_t - r_t^f]$	$\sigma(r_t - r_t^f)$
Boom	Becnhmark	0.063	0.208	0.010	0.005	0.053	0.208
	Target Date	0.018	0.151	0.010	0.007	0.008	0.151
	Free Access	0.027	0.132	0.013	0.007	0.014	0.132
Bust	Becnhmark	0.065	0.232	-0.011	0.004	0.076	0.231
	Target Date	0.015	0.140	-0.012	0.006	0.027	0.139
	Free Access	0.003	0.130	-0.010	0.006	0.013	0.130

Idiosyncratic Productivity Process

back

$Z = 0$

i, j	low	high	star	stationary distribution
low	0.6	0.4	0	0.104930
high	0.05	0.948625	0.001375	0.839444
star	0	0.02075	0.97925	0.055626
y	0.15	1	8.07	1.304083

$Z = 1$

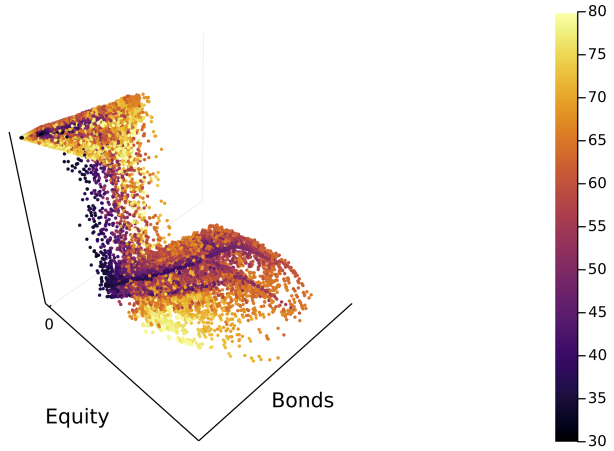
i, j	low	high	star	stationary distribution
low	0.3	0.7	0	0.031369
high	0.025	0.973625	0.001375	0.879255
star	0	0.02075	0.97925	0.089376
y	0.15	1	8.07	1.397591

normalization: 1.39

Algorithm back

1. initialize neural net parameters
2. simulate a long path for a large cross section of agents using policy functions and generalized moments $\Longrightarrow$ ergodic set of the economy
    1. randomly draw the distribution at a time from ergodic set
    2. simultaneously simulate for a lifetime
        1. a cross section of agents whose initial conditions are given by the drawn distribution
        2. spring up an AI as an individual agent at a^{enter}, the objective of whom is to maximize realized lifetime utility
    3. adjust individual agent's neural net parameters (not cross section neural nets)
    4. update cross sectional agent's neural nets
3. repeat until convergence

Basis Function $\mathscr{G}(x)$ GM



- Age important dimension of heterogeneity

Counterfactual: Target Date Economy - Sharpe Ratio

back

	Average Sharpe Ratio			
	By Age		By Wealth	
	<60	>60	<90%	>10%
Benchmark	0.040	0.151	0.046	0.294
Target Date	0.066	0.071	0.036	0.349

- Equalizing price of risk across cohorts
  - more equity holdings for the young who have large human capital
  - reduction in price of risk through re-distributing equity holdings across age is mostly individual effect

Counterfactual: Target Date Economy - Sharpe Ratio

back

	Average Sharpe Ratio			
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Benchmark	0.040	0.151	0.046	0.294
Target Date	0.066	0.071	0.036	0.349

- Equalizing price of risk across cohorts
  - more equity holdings for the young who have large human capital
  - reduction in price of risk through re-distributing equity holdings across age is mostly individual effect
- Little change in the price of risk across wealth distribution
  - for the poor: more equity holdings, but GE stabilizes equity return
  - for the rich: stabilized equity return but higher portfolio share in equity
  - compositional effect: shifting equity holdings to the poor

Counterfactual: Free Access Economy - Sharpe Ratio

back

	Average Sharpe Ratio			
	By Age		By Wealth	
	<60	>60	<90%	>10%
Benchmark	0.040	0.151	0.046	0.294
Target Date	0.066	0.071	0.036	0.349
Free Access	0.061	0.118	0.048	0.319

- Compared to the target date economy
  - Sharpe ratios are similar across subgroups
  - less redistribution to the young, more redistribution to the bottom 90%

Parameters from Literature back

Parameter	Value	Source
Aggregate state		
switching intensity	0.125	Krusell-Smith 1998
Firm		
capital share	0.36	Kydland-Prescott 1982
adjustment cost	1	Brunnermeier-Sannikov 2014
capital volatility	0.1	Brunnermeier-Sannikov 2014
depreciation	0.09, 0.11	Krusell-Smith 1998
average payout yield	0.049	Fernandez-Villaverde-Hurtado-Nuno 2022
Government		
income tax rate	0.2	De Nardi and Yang (2014)
government bond supply	0.3	SIFMA Research

Parameters from Literature and Data matrices back

Parameter	Value	Source
Households		
enter, retirement, death age	30,65,80	
age distribution		1998 US Mortality Database
mortality risk		1998 US Mortality Database
CRRA	10	
income age profile		Imrohoroglu-Imrohoroglu-Joines 1995
productivity type & transition		Den-Haan 2010 + Davila-Hong-Krusell-Rios-Rull 2012
social security	0.3	35% replacement rate
participation for high type	{0.5, 0.002}	stock market participation at age 30, 50
bequest arrival by type	{0, 0.05, 0.1}	

Capital structure and payout rules back

- Asset supply: firms as Merton investors with log utility
  - firms have assets $K_t = N_t + B_t$, $leverage_t = B_t/K_t$, capital to net worth $\omega_t = \frac{1}{1-leverage_t}$
  - maximize discounted log payouts: value maximization and payout smoothing
  - choose ω_t and payouts
  - trade-off: higher ω_t earns higher ER_t, payout smoothing lowers ω_t
    $\rightarrow$ interior solution for ω_t
- Optimal capital to net worth ratio $\omega_t \approx ER_t/\sigma^2$
- Optimal payout yield equals firm discount rate on average, varies over time due to adjustment cost

Capital structure and payout rules back

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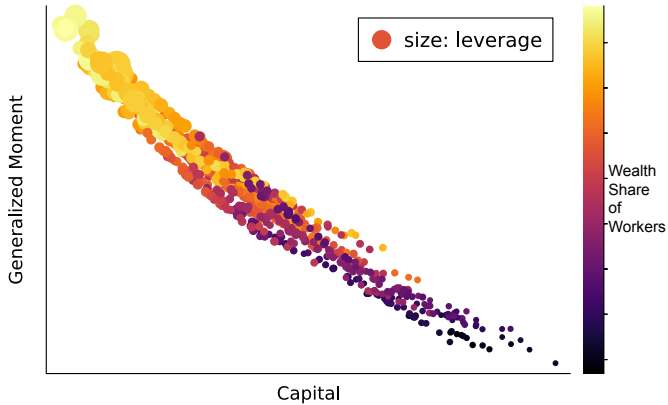
Generalized Moment and Capital Basis



- Using the first moment is not enough

Quantification: Targeted Aggregate Wealth Moments

	Wealth-income ratio	Retiree wealth share	Top 10% wealth share
Data	3.478	0.268	0.695
Benchmark	4.178	0.210	0.631

- Targeted parameters
  - household discount rate, bequest parameters

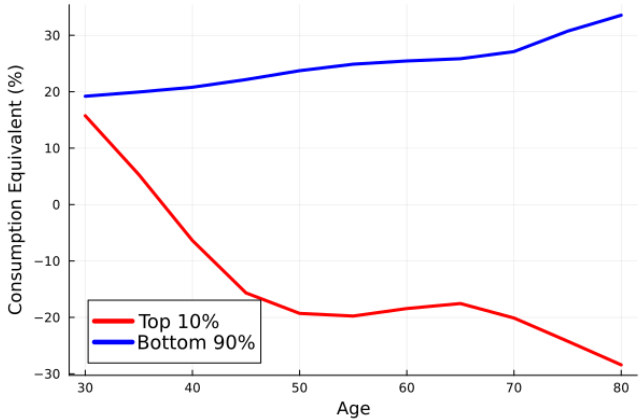
$$u^B(b) = \underline{b}\frac{(\bar{b}+b)^{1-\gamma}}{1-\gamma}$$

  - GMM estimation is exactly identified

Capital structure and payout rules back

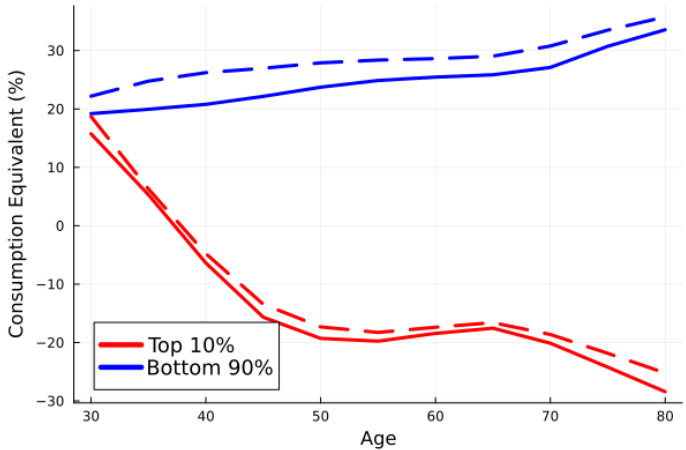
- a continuum of homogeneous firms issue bonds to finance capital investment
- balance sheet: $K_t = N_t + B_t$
- leverage: B_t/K_t, capital to net worth: $\omega_t = \frac{1}{1-leverage_t}$
- expected excess return on capital: $ER_t = MPK_t - r_t^f - \Phi(\iota_t) - \delta(Z_t)$
- equity return: $dN_t = \left(\left[r_t^f + \omega_t ER_t\right] N_t - D_t\right) dt + \sigma \omega_t N_t dW_t$
- optimal capital structure: $\omega_t = \frac{1}{1-leverage_t} \approx ER_t/\sigma^2$
- optimal payout yield equals firm discount rate on average, varies over time due to adjustment cost
- micro-foundation: maximize discounted log payouts

Counterfactual: Target Date Economy - Welfare by Wealth



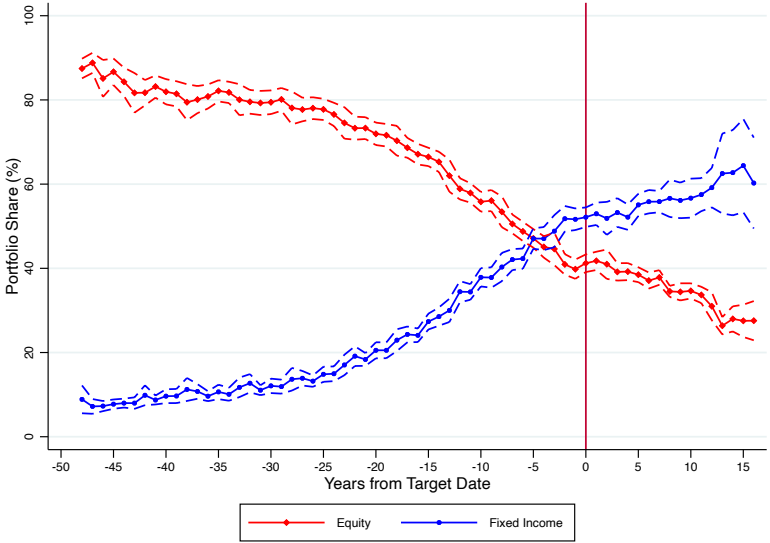
- Bottom 90%: increased participation, accumulate more wealth
- Top 10% lose, especially at old ages: reduction in equity premium 6% to 2%
- Young agents thin in buffer stock: stabilized equity returns

Counterfactual: Free Access Economy - Welfare by Wealth

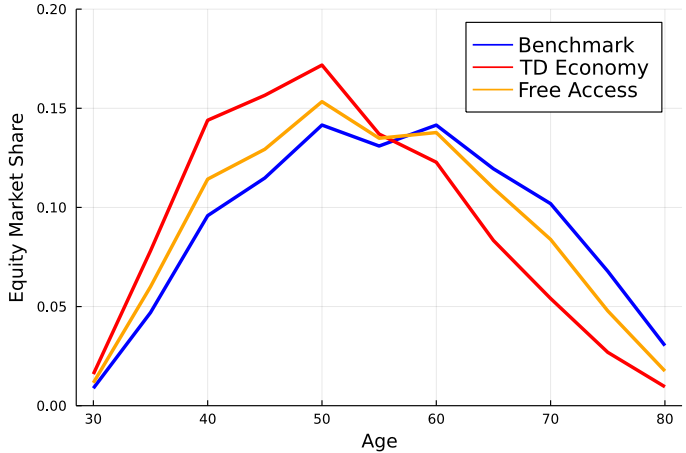


- $< 5\%$ further improvements
- GE changes stocks as an asset, deviations from optimal portfolio less important

Target Date Funds - Glide Path

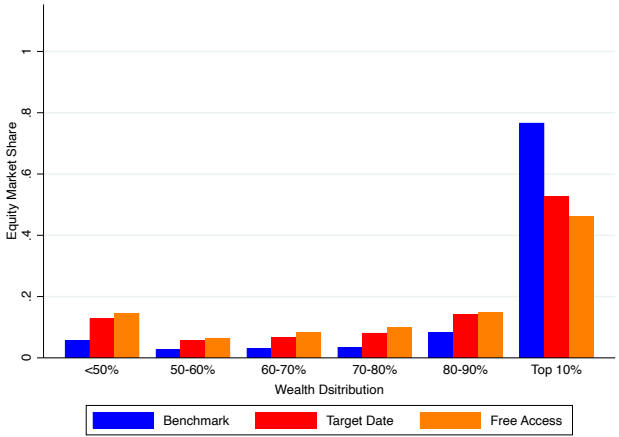


Counterfactual: More Equity Holdings for the Young



- Age 55 and less: 53% (benchmark), 70% (target date), 60% (free access)
- Better risk sharing: the young are rich in relatively safe human capital

Counterfactual: Reduction in the Concentration of Equity Holdings

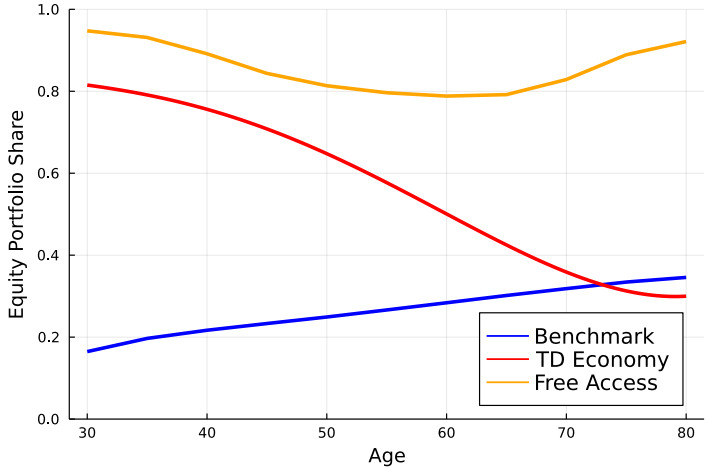


- Top 10% equity market share: 77% (benchmark), 53% (target date), 46% (free access)
- Better risk sharing: top 10% more exposed to aggregate risk, high risk compensation

Counterfactual: Target Date Economy - Asset Prices cd

- Equity premium smaller: 6.4% to 1.7%
  - product of stock return volatility and Sharpe ratio
- Equity returns are less volatile: 21.9% to 14.6%
  - more elastic stock demand: trade against market outcomes
- Sharpe ratio down: 0.29 to 0.12
  - age 55 and less: 53% (benchmark), 70% (target date)
  - top 10% equity market share: 77% (benchmark), 53% (target date)
- Free access economy: same direction, further reductions
  - more elastic stock demand: trade against market outcomes
  - redistribution of equity holdings leads to better risk sharing

Counterfactual: Portfolio Age Profile



- Stocks outperform bonds, driving up portfolio share in equities
- Bequest and drawing down financial savings